



第二届旋转与磁场下的QCD物质研讨会

QCD phase transition at finite isospin density and magnetic field

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[GC, et. al. , NST \(2026\) ; arXiv:2608.21996 ; Phys. Rev. D \(2021, 2023, 2024\).](#)

2026.9.12 @ Anhui University of Science and Technology



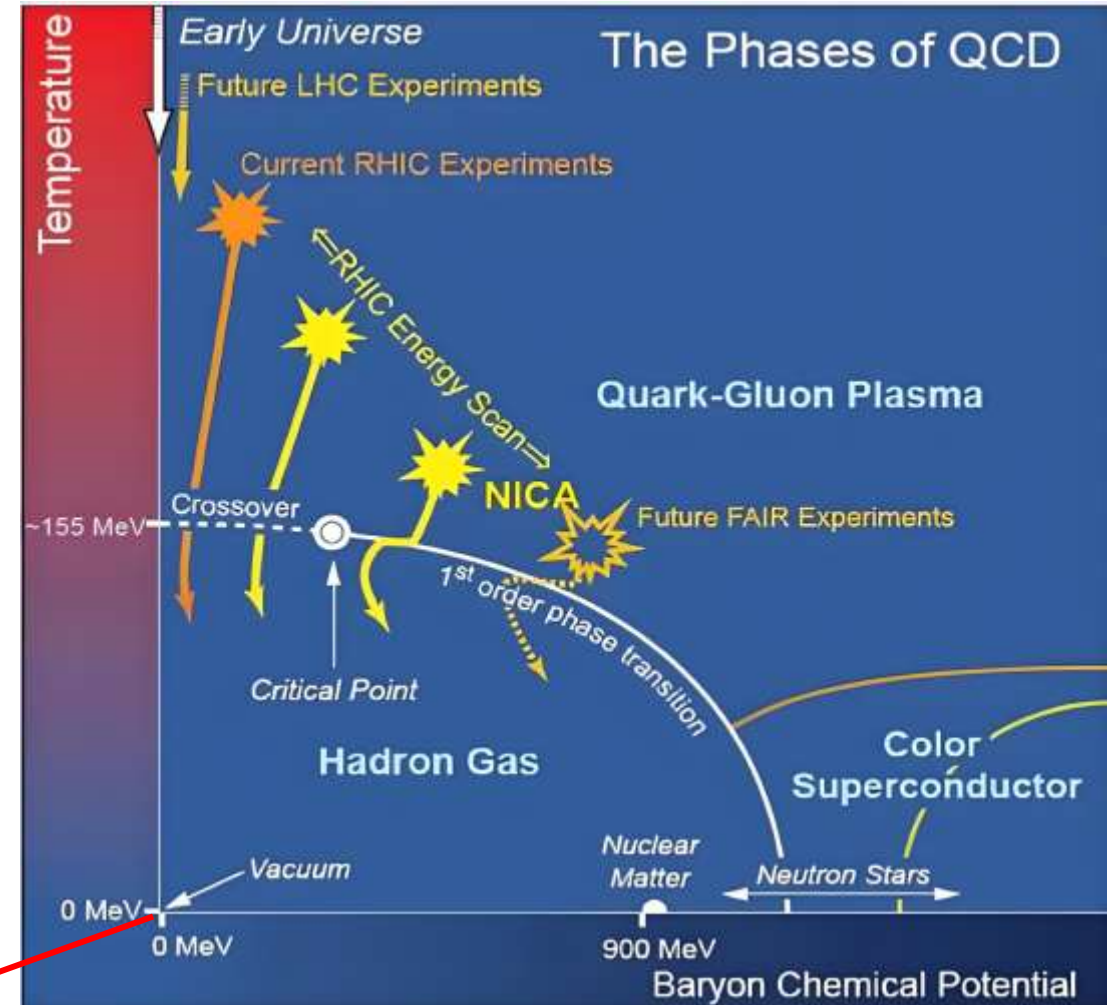
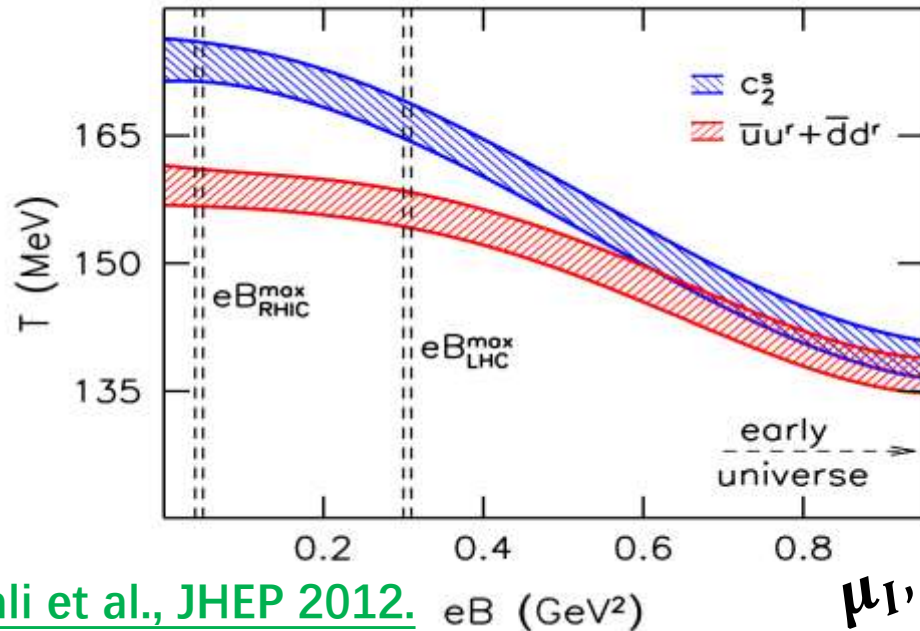
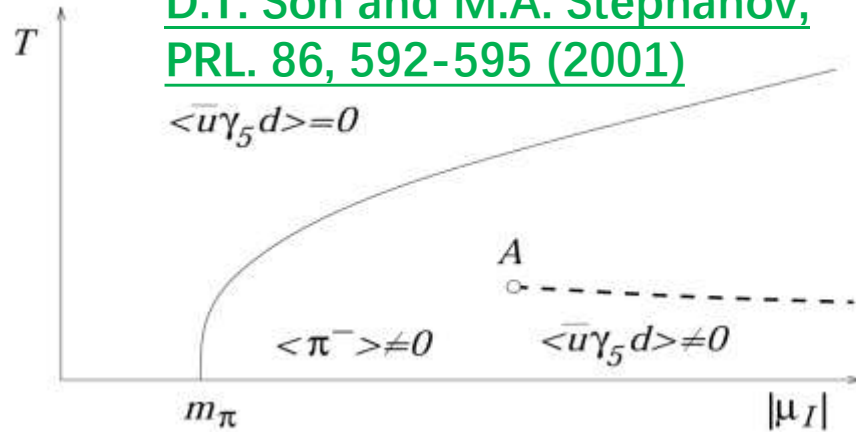
Outline

- ◆ QCD phase transition under new circumstances
- ◆ Newest LQCD results and an important indication
- ◆ Previous results at finite μ_I and B
- ◆ Two-flavor NJL model study
- ◆ Three-flavor NJL model study
- ◆ Summary and prospective



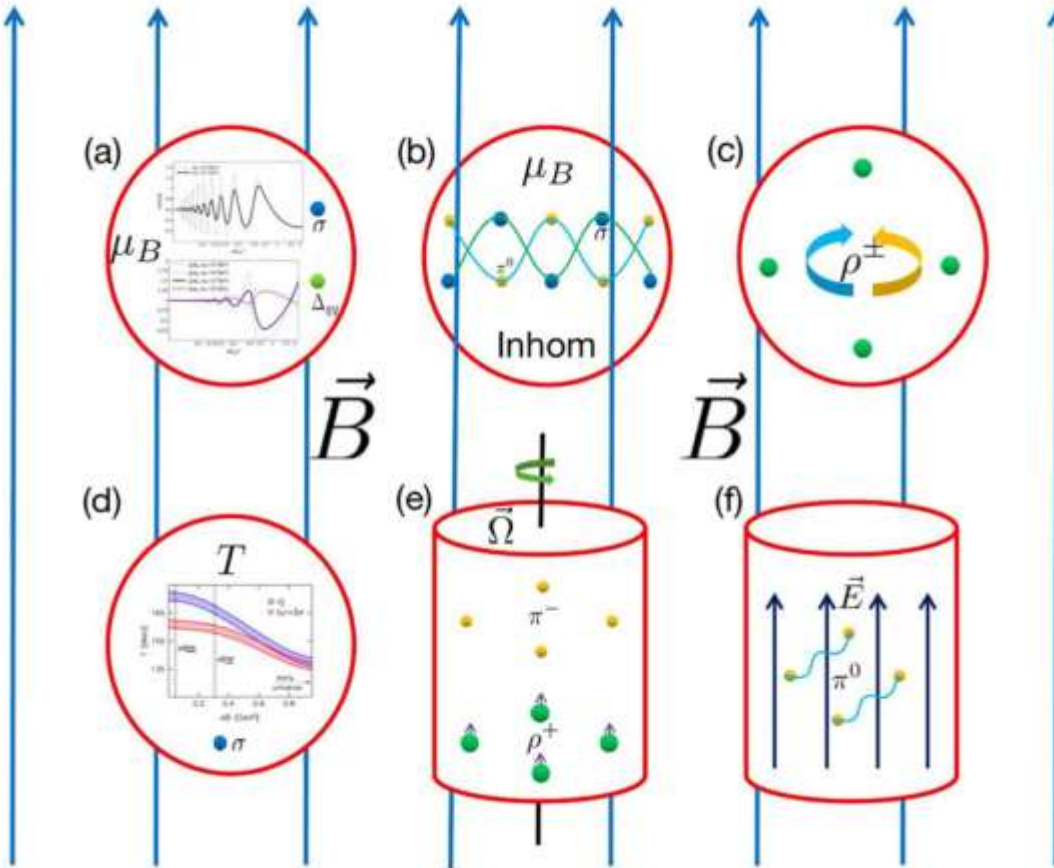
QCD phase transition under new circumstances

D.T. Son and M.A. Stephanov,
PRL. 86, 592-595 (2001)



G. Bali et al., JHEP 2012.

$\mu_1, B, E, \Omega \dots$



[GC, Eur. Phys. J. A \(2021\) 57:264](#)

1. μ_B, B :
de Haas–van Alphen oscillation,
inhomogeneous,

2. Ω, B : $\pi^\pm, \rho^\pm$ condensation,

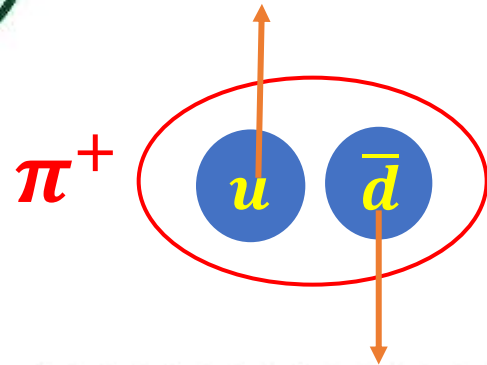
3. E, B : π^0, K^0 ... condensation,

4. Ω, μ_I : $\pi^\pm, \rho^\pm$ condensation,

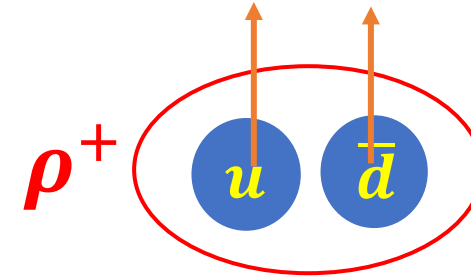
[H. Zhang, D. Hou and J. Liao,
Chin. Phys. C 44, 111001 \(2020\)](#)



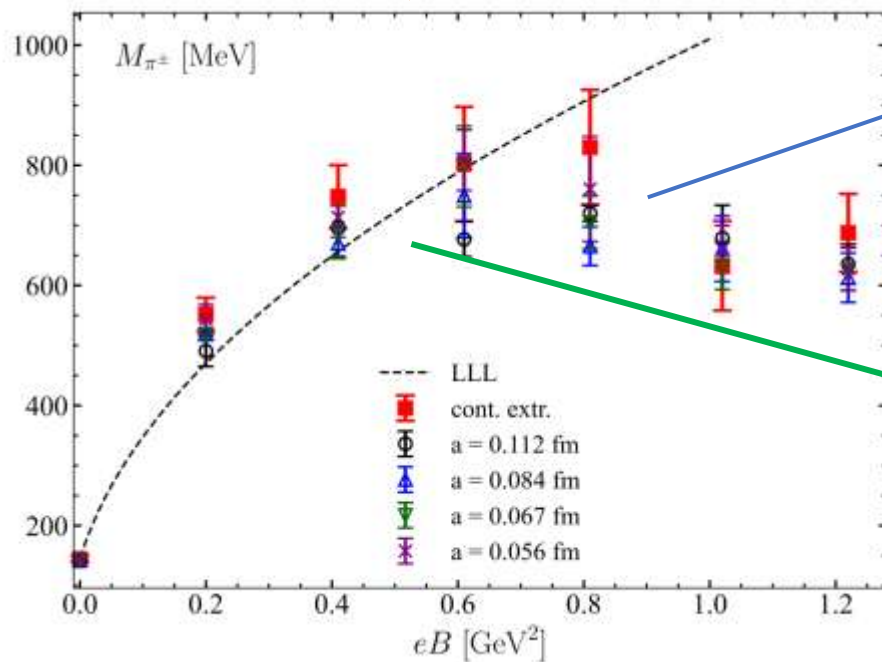
Newest LQCD results and an important indication



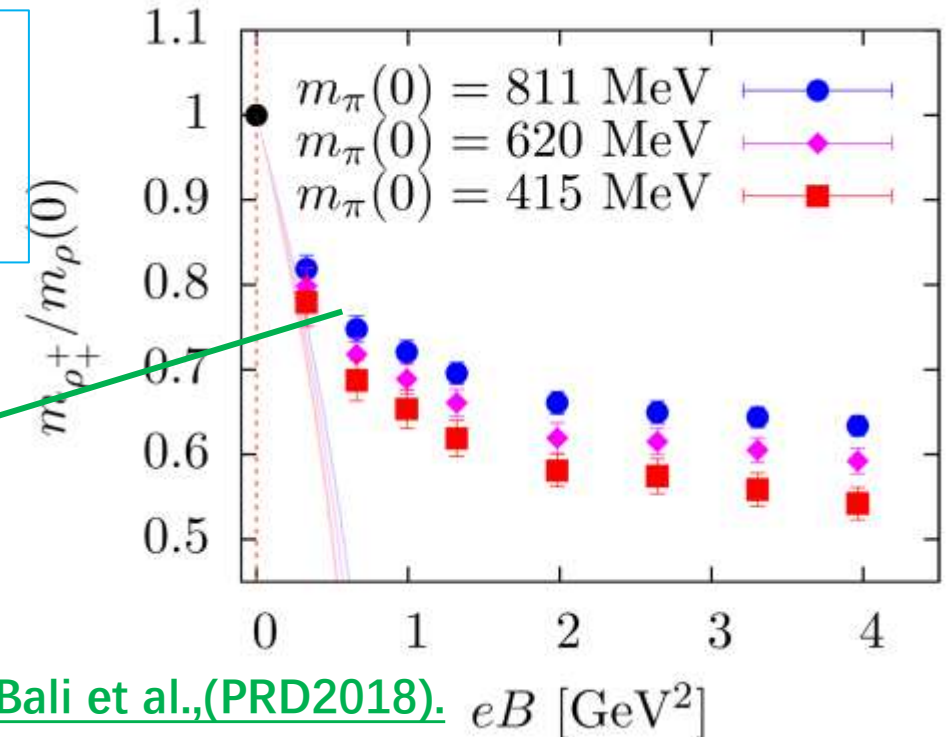
unstable with
both Ω and B



stable with
both Ω and B



reversal

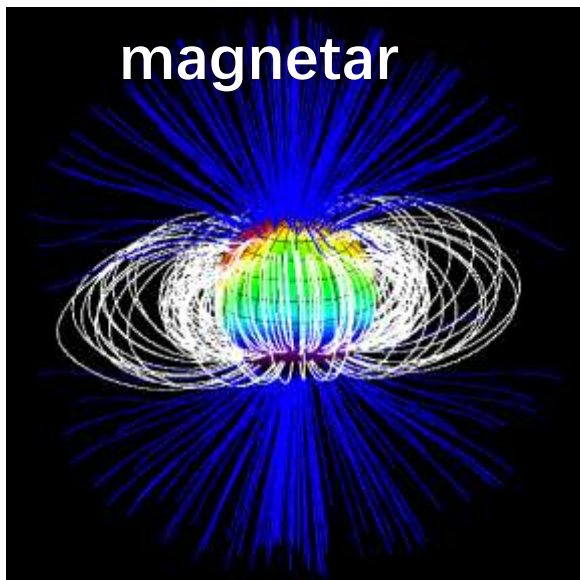


H.T. Ding and D. Zhang, arXiv: 2601.1835

G. Bali et al.,(PRD2018). $eB \text{ [GeV}^2\text{]}$



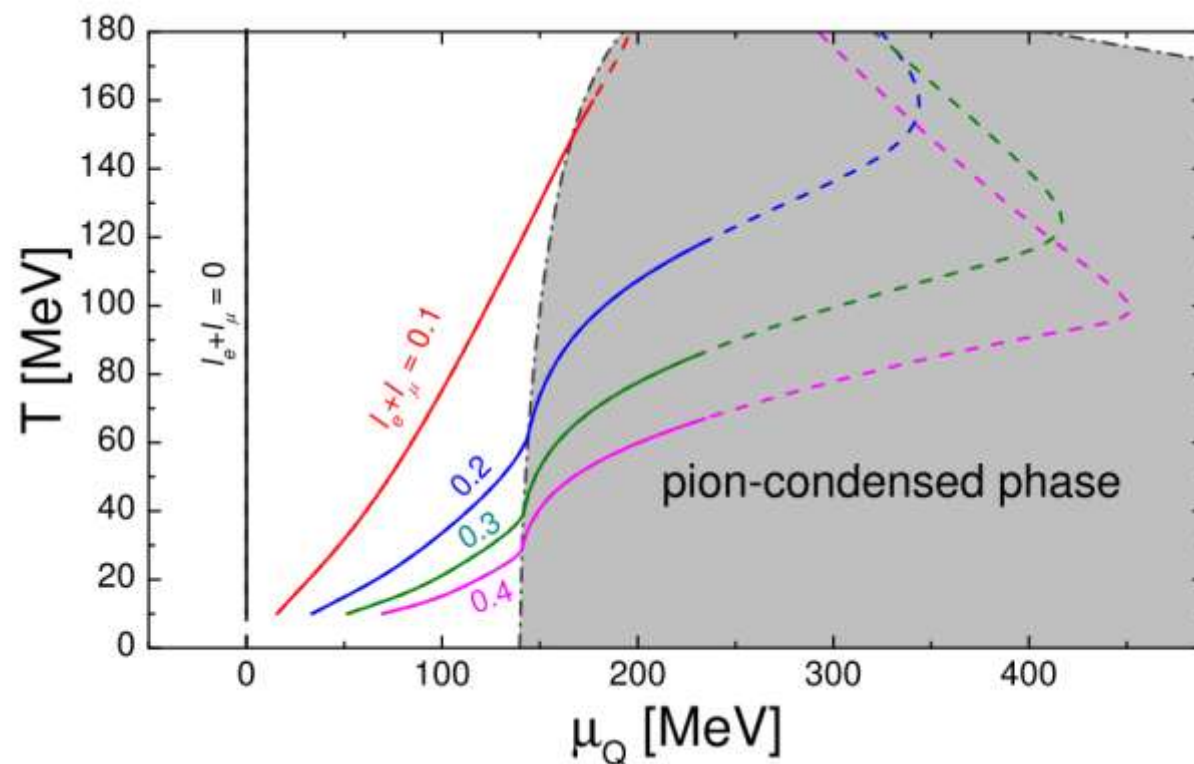
Relevant physical systems



magnetar



Big bang



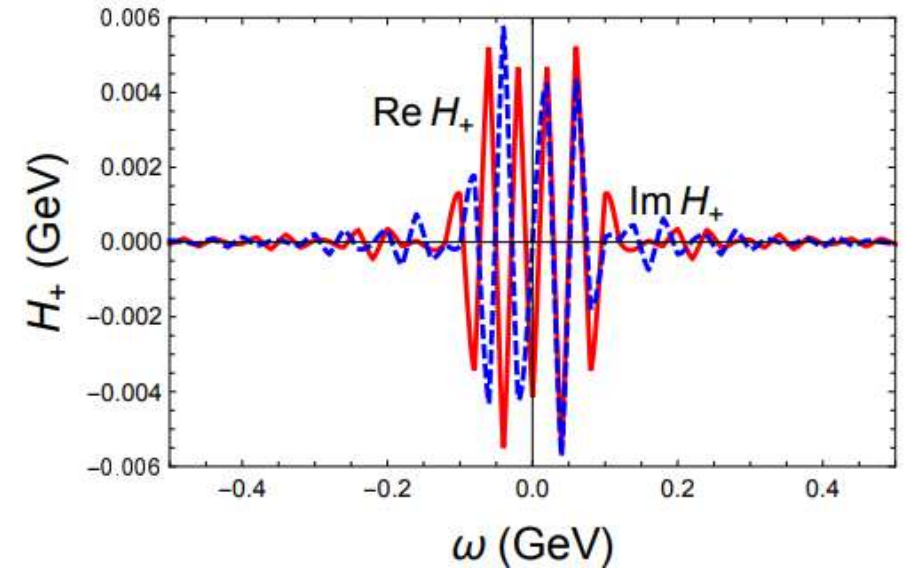
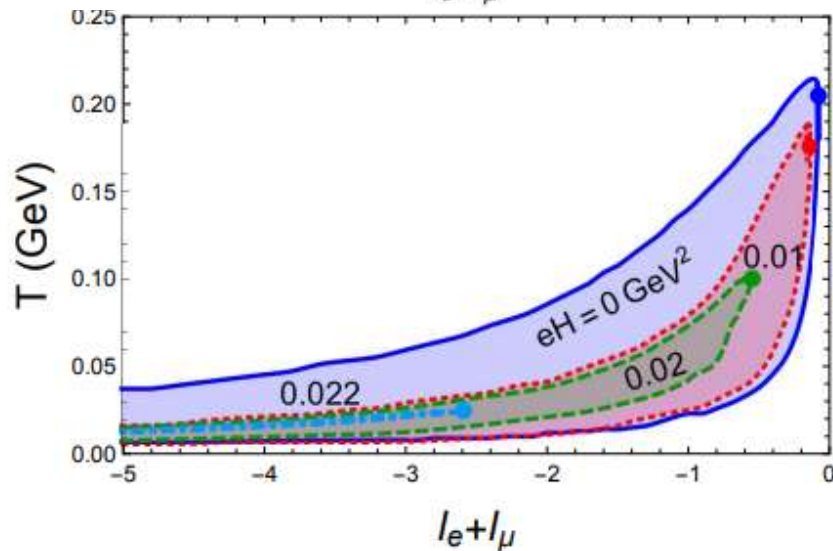
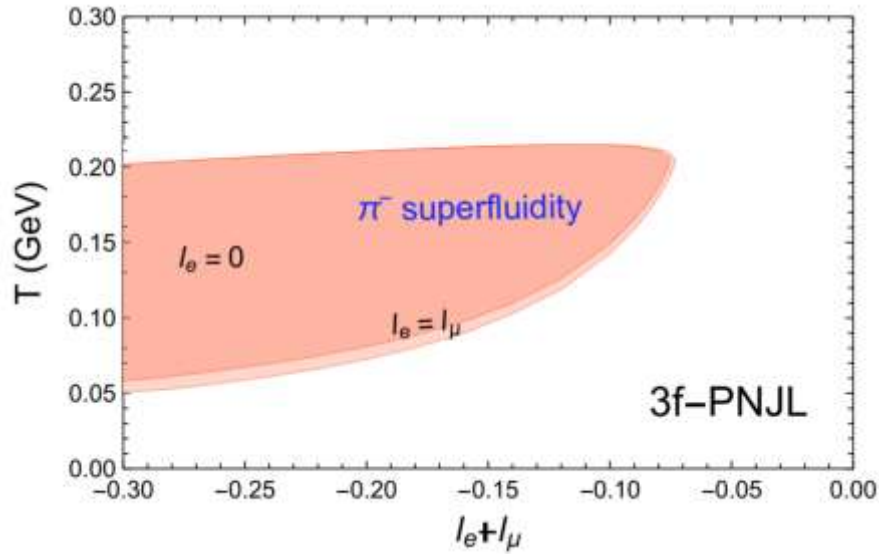
$$l = 0, Q = 0$$



V. Vovchenko et al., Phys. Rev. Lett. 126, 012701 (2021)



Previous results at finite μ_I and B



$\omega_c \sim 0.1 \text{ GeV}$, consistent with Λ_{QCD}

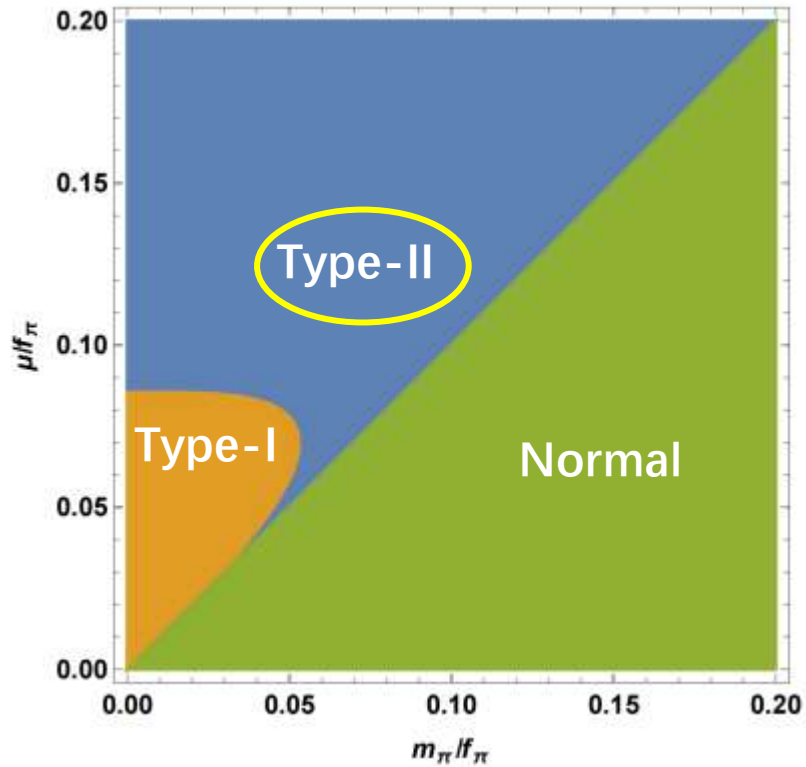


Relic GW: $f_c \sim 10^9 - 10^{10} \text{ Hz}$
or $0.1 - 1 \text{ K}$

GC, Phys. Rev. D 107, 014021 (2023).



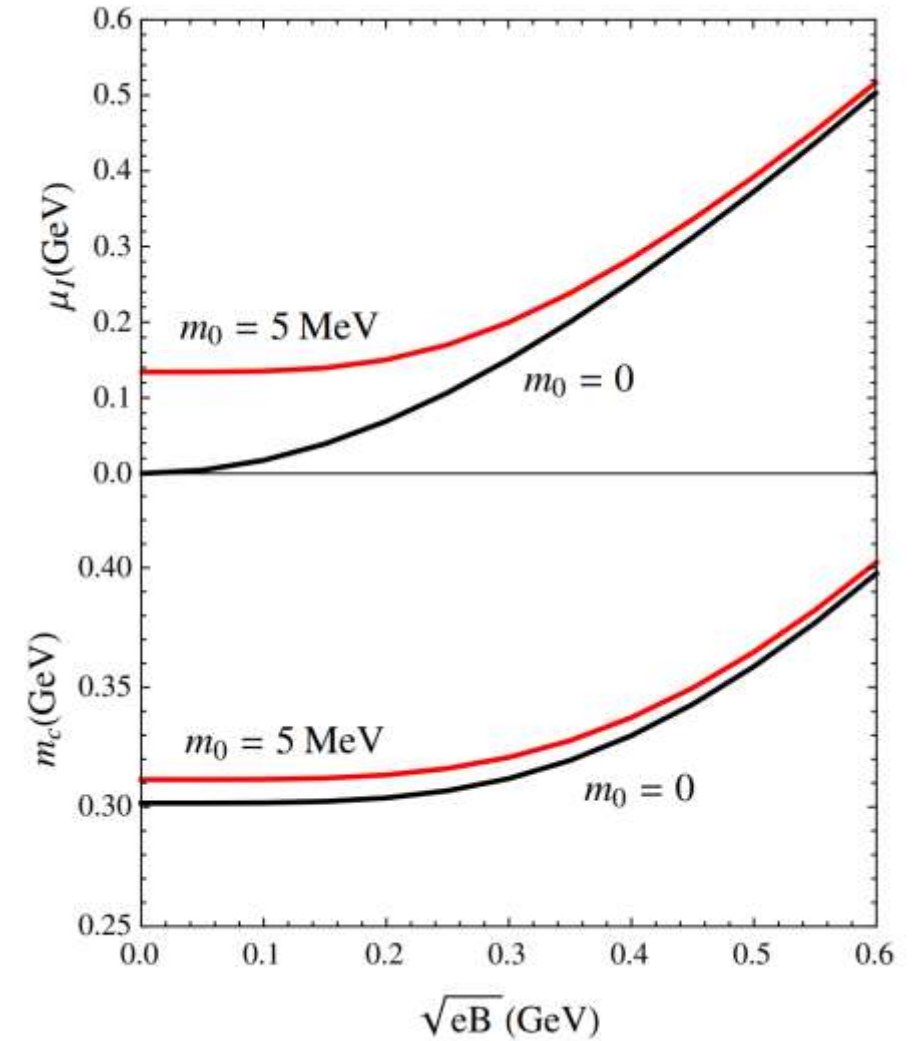
Previous results at finite μ_I and B



χ PT

$$\left\{ \begin{array}{l} m_A = q f_\pi \left(1 - \frac{m_\pi^4}{\mu_I^4} \right) \\ m_\theta = \mu_I \sqrt{\left(1 - \frac{m_\pi^4}{\mu_I^4} \right)} \end{array} \right.$$

[P. Adhikari et. al., Phys. Rev. C 91 \(4\) \(2015\) 045202.](#)



[GC and P. Zhuang, 92, 105030 \(2015\)](#)



I. Two-flavor NJL model

Lagrangian

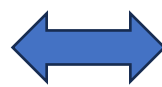
$$\mathcal{L} = \bar{\psi} \left(i\gamma^\mu D_\mu - m_0 + \frac{\mu_I}{2} \gamma^0 \tau_3 \right) \psi + G_S [(\bar{\psi}\psi)^2 + \underline{(\bar{\psi} i\gamma_5 \boldsymbol{\tau} \psi)^2}] - G_V [\underline{(\bar{\psi} \gamma^\mu \boldsymbol{\tau}^a \psi)^2} + (\bar{\psi} i\gamma^\mu \gamma_5 \boldsymbol{\tau}^a \psi)^2],$$

Hubbard-Stratonovich transformation

$$\mathcal{L} = \bar{\psi} \left[i\gamma^\mu \tilde{D}_\mu - m_0 - \sigma + \frac{\mu_I}{2} \gamma_0 \tau_3 - i\gamma_5 (\tau_3 \pi^0 + \tau_\pm \pi^\pm) \right] \psi - \frac{\sigma^2 + (\pi^0)^2 + \underline{\pi^\mp \pi^\pm}}{4G_S} + \frac{(\omega^\mu)^2 + (\rho_0^\mu)^2 + \underline{\rho_\mu^\mp \rho^{\pm\mu}} + (A^{a\mu})^2}{4G_V}$$

Inverse propagator

$$D_{\bar{V}_\mu \bar{V}_\mu}^{-1}(p) \equiv \frac{1}{2G_V} + \Pi_{\bar{V}_\mu \bar{V}_\mu}(p)$$



GL coefficient

$$\mathcal{A}_{\bar{V}_\mu \bar{V}_\mu} = \frac{1}{4G_V} + \frac{\Pi_{\bar{V}_\mu \bar{V}_\mu}(0)}{2}$$



Self-energy without Schwinger phase

Self-energy $\Pi_{\bar{V}_\mu \bar{V}_\mu}(p) = \int \frac{d^4 \mathbf{k}}{(2\pi)^4} \text{tr } \mathcal{S}(k) \Gamma_{\bar{V}_\mu^*} \mathcal{S}(p+k) \Gamma_{\bar{V}_\mu}$

Schwinger representation

$$S_f^E(\omega_n, \mathbf{k}) = -i \int_0^\infty ds e^{-s(m^2 + \omega_n^2 + k_3^2 + \mathbf{k}_\perp^2 \tanh B_f^s / B_f^s)} \xrightarrow{\quad} \boxed{\omega_n + i\mu_I/2}$$

$$(-k + m - i(k_{12} - k_{21}) \tanh B_f^s)(1 - i\gamma_1 \gamma_2 \tanh B_f^s)$$

Landau level representation — suitable for large μ_I

$$S_f^E(k) = -i \exp\left(-\frac{\mathbf{k}_\perp^2}{|q_f B|}\right) \sum_{n=0}^{\infty} (-1)^n \frac{D_n(q_f B, k)}{\left(k_4^f\right)^2 + k_3^2 + m^2 + 2|q_f B|n} \xrightarrow{\quad} \boxed{k_4^{u/d} \equiv \omega_n \pm i\frac{\mu_I}{2}}$$



Landau-energy regularization

$$I_{\pi^+, \perp} = \frac{eB}{3\pi} \sum_{n_1, n_2=0}^{\infty} (-1)^{n_1+n_2} \left\{ (k_3^2 + k_4^u k_4^d + m^2) \left(\frac{1}{n_2} - \frac{1}{n_1} \right) - 2\frac{eB}{3} \right\} C(n_1, n_2)$$

$$\Delta\Pi_{\pi^+\pi^+}^{B, \mu_I} = -\frac{2N_c}{3\pi} \left[\sum_{n_1, n_2=0}^{N_1, N_2} g_{\pi}(eB, n_1, n_2) - \sum_{n'_1, n'_2=0}^{N'_1, N'_2} g_{\pi}(eB', n'_1, n'_2) \right]$$

Two schemes

$$N_1 = N_2 = N'_1 = N'_2$$

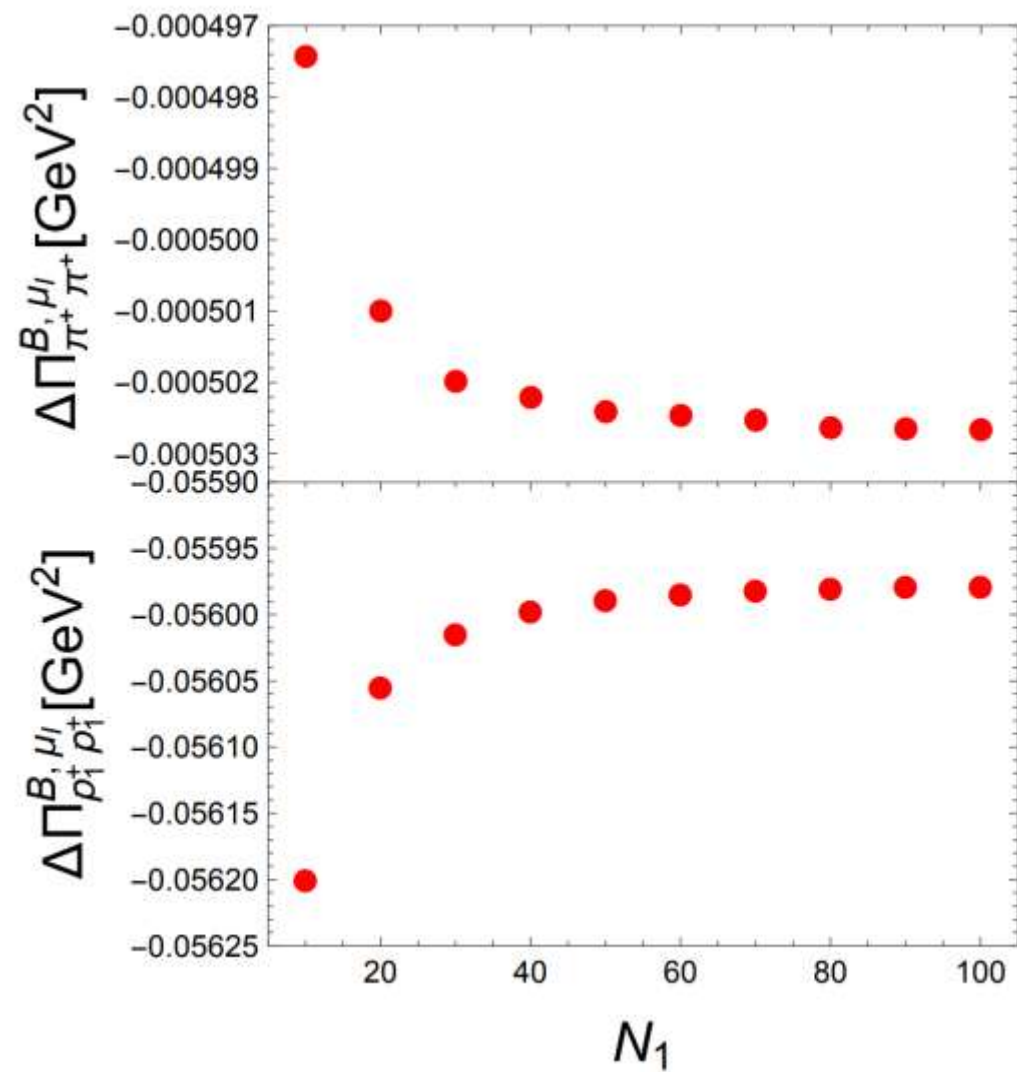
divergent

$$|q_u B n_1|, |q_d B n_2|, |q_u B' n'_1|, |q_d B' n'_2| \leq \Lambda^2$$

More physical

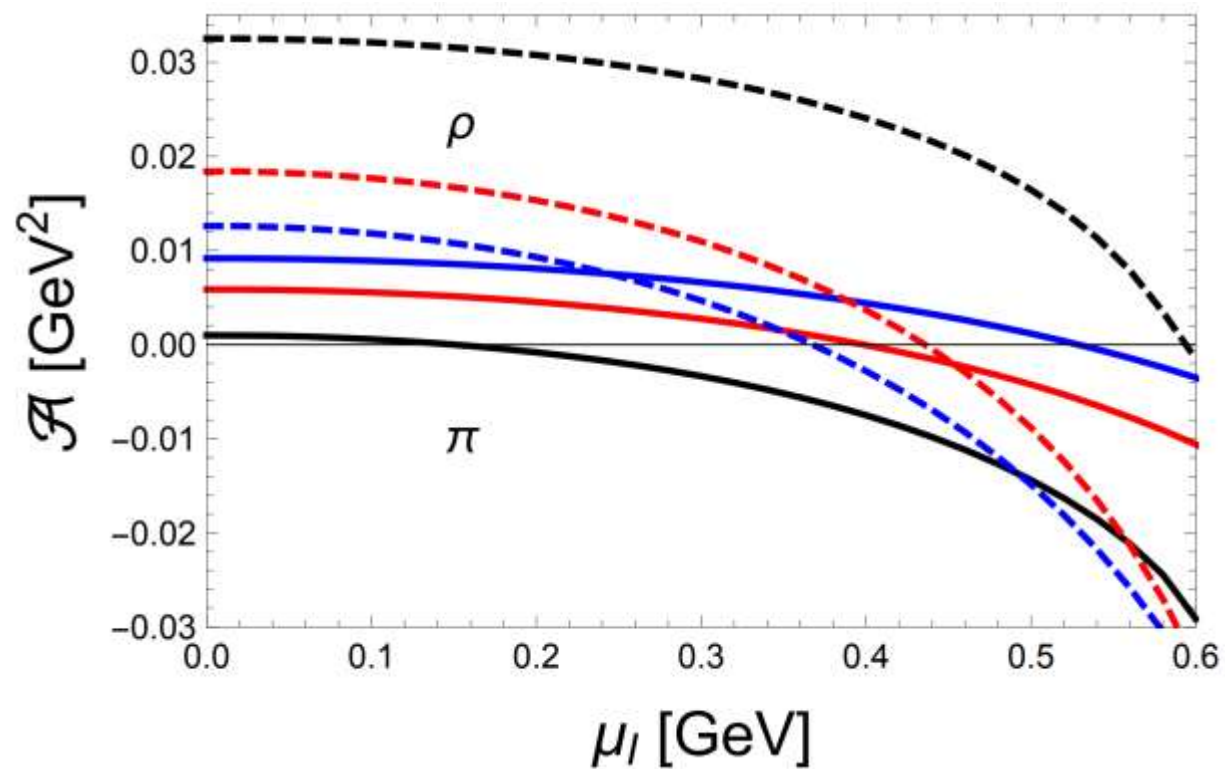
convergent

$$\mathcal{A}_{\pi^+\pi^+} = \frac{1}{4G_S} - 2N_c \int^{\Lambda} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{E_{\mathbf{k}}}{E_{\mathbf{k}}^2 - (\mu_I/2)^2} + \Delta\Pi_{\pi^+\pi^+}^B + \Delta\Pi_{\pi^+\pi^+}^{B, \mu_I} + \Pi_{\pi^+\pi^+}^{B, \mu_I, T}$$

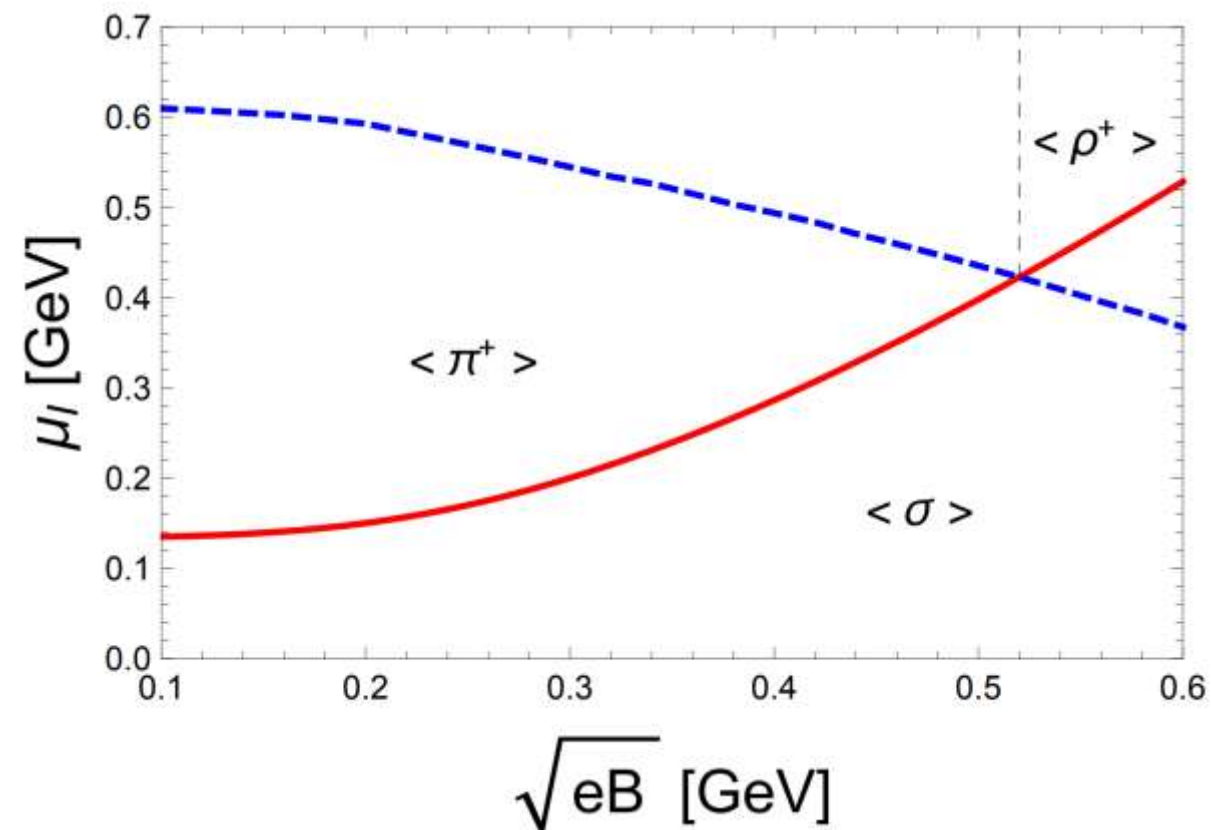




Numerical results



$\sqrt{eB}=0$
 0.2 GeV
 0.5 GeV





II. Three-flavor NJL model

Bosonized Lagrangian

$$\mathcal{L} = \bar{\psi} \left(i \not{D} - m_0 + \frac{\mu_I}{2} \gamma^0 \lambda^3 - (S^a + i\gamma_5 P^a - V^a - i\gamma_5 A^a) \lambda^a \right) \psi \\ - \frac{(S^a)^2}{4G_{aa}^-} - \frac{(P^a)^2}{4G_{aa}^+} + \frac{V^{a\mu} V_\mu^a + A^{a\mu} A_\mu^a}{4G_V} + \dots$$

$$\pi^+(x) \bar{u}(x) i\gamma_5 d(x) \left\{ \begin{array}{l} \pi^+(x) = \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot x} \pi^+(k) \quad \text{Vacuum-like} \\ \pi^+(x) = \sum_{n=0} \int \frac{d^3 \tilde{k}}{(2\pi)^3} e^{-i\tilde{k} \cdot x} E_n \left(y + \frac{k_1}{|eB|} \right) \pi_n^+(k_{||}) \end{array} \right.$$

Landau space

$$\pi_n^+(x) \bar{u}_{n_1}(x) i\gamma_5 d_{n_2}(x) \left\{ \begin{array}{l} \pi_n^+(x) \rightarrow e^{-ie \int_{x_0}^x \Delta A \cdot dz} \pi_n^+(x), \\ \bar{u}_{n_1}(x) \rightarrow e^{iq_u \int_{x_0}^x \Delta A \cdot dz} \bar{u}_{n_1}(x), \\ d_{n_2}(x) \rightarrow e^{-iq_d \int_{x_0}^x \Delta A \cdot dz} d_{n_2}(x) \end{array} \right.$$

Gauge invariant



l -independence in symmetric gauge

$$A_\mu = (0, B y/2, -B x/2, 0) \quad \longrightarrow \quad M_{n,l}(q_\parallel, x) \equiv e^{-i q_\parallel \cdot x_\parallel} \chi_n^l(eB, x_\perp)$$

$$\Pi_{n,l}(q_3, q_4) \equiv -\frac{1}{V_4} \int d^4x d^4x' e^{-i[q_3(x_3-x'_3)+q_4(x_4-x'_4)]} \chi_n^{l*}(eB, x_\perp) \chi_n^l(eB, x'_\perp) \text{Tr} G_u(x, x') \Gamma_M^\dagger G_d(x', x) \Gamma_M$$

Landau level representation

[V. A. Miransky and I. A. Shovkovy, Phys. Rept. \(2015\).](#)

$$\Pi_{n,l}^\perp \equiv - \int d^2x_\perp d^2x'_\perp \chi_n^{l*}(eB, x_\perp) \chi_n^l(eB, x'_\perp) e^{i\Phi(eB, x_\perp, x'_\perp) - \frac{|eB|}{4} \Delta \mathbf{x}_\perp^2} \sum_{n_u, n_d=0}^{\infty} f_{n_d, q_d}^{n_u, q_u}(\Delta x_1, \Delta x_2)$$

$$f_{n_d, q_d}^{n_u, q_u}(\Delta \mathbf{x}_\perp^2) = \sum_{n=0}^{n_u+n_d} C(n_u, q_u; n_d, q_d; n, e) L_n \left(\frac{|eB|}{2} \Delta \mathbf{x}_\perp^2 \right) \quad \text{Laguerre polynomial}$$

$$\begin{aligned} & \frac{|eB|}{2\pi} e^{i\Phi(eB, x_\perp, x'_\perp) - \frac{|eB|}{4} \Delta \mathbf{x}_\perp^2} L_n \left(\frac{|eB|}{2} \Delta \mathbf{x}_\perp^2 \right) \\ &= \sum_{l=-n}^{\infty} \chi_n^{l*}(eB, x'_\perp) \chi_n^l(eB, x_\perp) \end{aligned} \quad \longrightarrow \quad \Pi_{n,l}^\perp = \frac{2\pi}{|eB|} \sum_{n_u, n_d=0}^{\infty} C(n_u, q_u; n_d, q_d; n, e) \Theta(n_u + n_d - n)$$



Meson propagators for different Landau levels

$$D_M^{-1}(q_{\parallel}, n) = \frac{1}{4G} + \frac{|q_u B|}{4\pi} \frac{|q_d B|}{4\pi} \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \sum_{n_u=0}^{\infty} \sum_{n_d=0}^{\infty} \frac{N(n, n_u, n_d; eB, q_u B, q_d B)}{\left[(k_4^u + q_4)^2 + E_{n_u}^2 (k_3 + q_3) \right] \left[(k_4^d)^2 + E_{n_d}^2 \right]}$$

$$h(\alpha, n, n_u, n_d) = \frac{16\pi}{|eB|} \left(\frac{2}{|eB|} \right)^{\alpha} \int_0^{\infty} dr e^{-r} r^{\alpha} L_n(r) L_{n_u}^{\alpha} \left(\frac{2}{3} r \right) L_{n_d}^{\alpha} \left(\frac{1}{3} r \right)$$

$$\rightarrow h(\alpha, 0, n_u, n_d) = \frac{16\pi}{|eB|} \left(\frac{2}{|eB|} \right)^{\alpha} \frac{(n_u + n_d + \alpha)!}{n_u! n_d!} \left(\frac{1}{3} \right)^{n_u} \left(\frac{2}{3} \right)^{n_d}$$

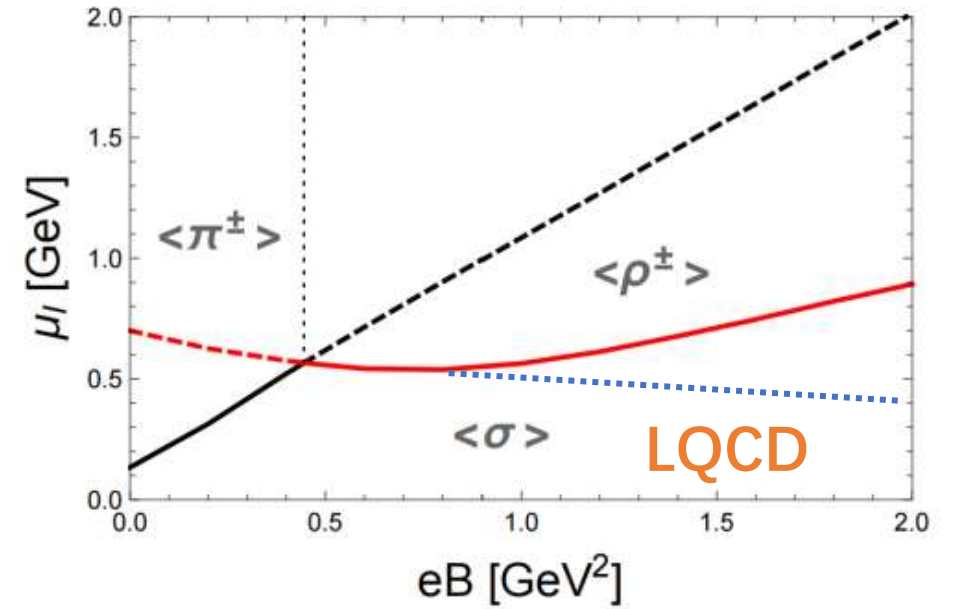
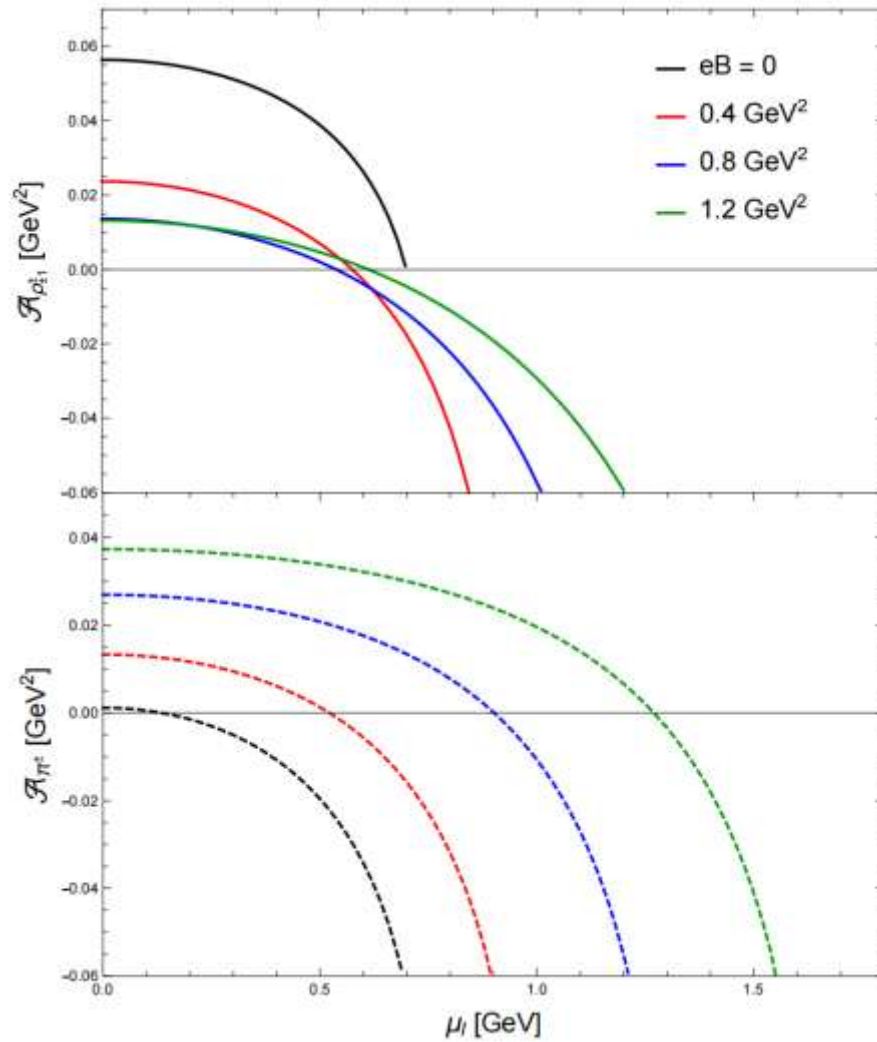
$$D_{\pi^{\pm}}^{-1} = \frac{1}{2G_{11}^{+}} - 2N_c \int d^4 \Delta x e^{-i(q_3 \Delta x_3 + q_4 \Delta x_4)} P_n(\Delta \mathbf{x}_{\perp}) \text{tr} [\tilde{G}_u(\Delta x) i\gamma_5 \tilde{G}_d(-\Delta x) i\gamma_5]$$

Regularization for a given Landau level

$B \rightarrow 0$



Numerical results

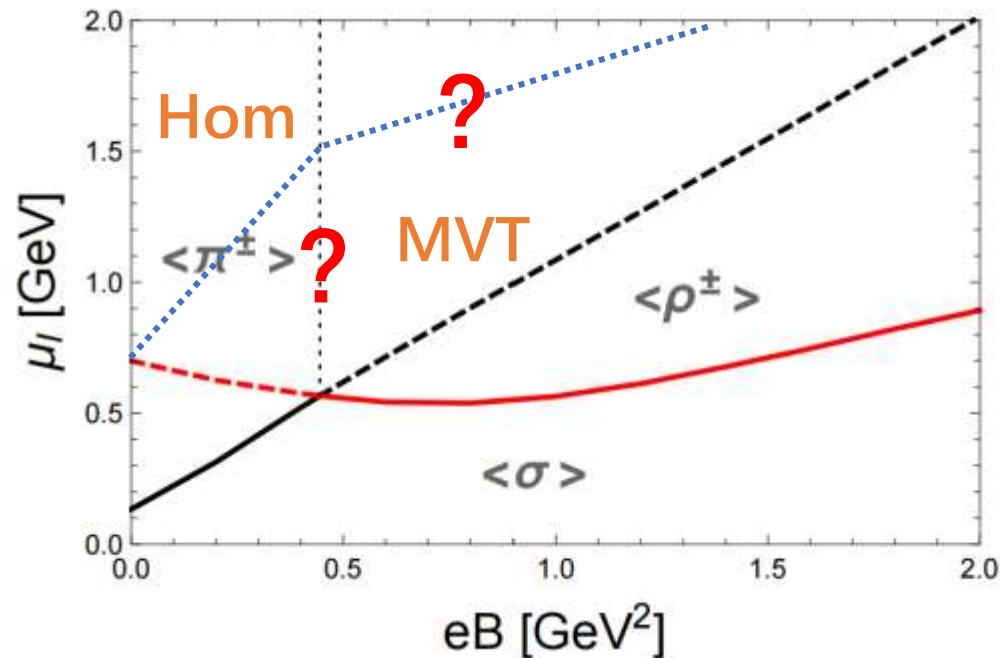


$\pi^\pm$ mix with $\rho_0^\pm$ but not $\rho_{\pm 1}^\pm$



Summary and prospective

- ◆ Projecting to the **Landau eigenstates** for $\pi^\pm$ and $\rho^\pm$
- ◆ **Landau representation** for finite μ_I and B
- ◆ **rho superconductor** at **larger B** due to **energy reversal**



Thanks!